HOME WORK VIII, DIFFERENTIAL GEOMETRY: ISOMETRIC IMMERSIONS, HOPF-RINOW THEOREM, SPACES OF CONSTANT SECTIONAL CURVATURE.

Due April 18. The Home Work must be uploaded on Canvas as a pdf. In addition to the class notes, you may find helpful Do Carmo's Riemannian geometry (Chapters 6,7,8). Please contact me if you have any questions!

1. Write out the Codazzi equation and the Ricci equation in the partial case of the space of constant sectional curvature.

2. Fix $p \in M$. Let $\sigma \subset T_pM$ be two-dimensional. Prove that the Gauss curvature at p of the two dimensional image of an open set $U \subset \sigma$, such that $p \in U$, under the exponential map $\exp_p$, equals the sectional curvature of σ.

3. Let M be a totally geodesic sub-manifold of $\bar{M}$. Let X, Y belong to T_pM. Show that $\bar{\nabla}_X Y = \nabla_X Y$.

4. Give an example of a differentiable Riemannian manifold, which is not geodesically complete, and there are two points in it which cannot be joined by a minimal geodesic. (thereby, you will show that the assumption of geodesic completeness in Hopf-Rinow theorem is necessary.)